Published by Institute of Physics Publishing for SISSA/ISAS
Received: *February 15, 2002*
Accepted: *June 5, 2002*

Boundary multi-trace deformations and OPEs in AdS/CFT correspondence

Anastasios C. Petkou
CERN Theory Division
CH-1211 Geneva 23, Switzerland
E-mail: tassos.petkou@cern.ch

Abstract: We argue that multi-trace deformations of the boundary CFT in AdS/CFT correspondence can arise through the OPE of single-trace operators. We work out the example of a scalar field in AdS_5 with cubic self interaction. By an appropriate reparametrization of the boundary data we are able to deform the boundary CFT by a marginal operator that couples to the conformal anomaly. Our method can be used in the analysis of multi-trace deformations in $\mathcal{N} = 4$ SYM where the OPEs of various single-trace operators are known.

Keywords: AdS-CFT and dS-CFT Correspondence, Conformal and W Symmetry, Extended Supersymmetry.

Contents

1. Introduction

The AdS/CFT correspondence [1] is a concrete realization of the broad class of holographic ideas. An implication of such ideas is that the classical equations of motion of fields that live in a curved bulk space determine the quantum states of a conformal field theory (CFT) that lives in the boundary. In the prototype example of the AdS/CFT correspondence, the equations of motion of classical IIB supergavity on $\mathrm{AdS}_5\times\ \mathrm{S}^5$ determine the correlation functions of $\mathcal{N}=4$ SYM on $\mathbb{R}^4$.

A remarkable success of the above example of $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence is the equivalence of (part of) the bulk and boundary spectra [2]. For example, the Kaluza-Klein modes of IIB SUGRA on $\mathrm{AdS}_5\times\ \mathrm{S}^5$ are in one-to-one correspondence with chiral gauge invariant operators of $\mathcal{N}=4$ SYM, which are realised as single-traces of its elementary fields. However, it is also clear that the boundary CFT spectrum contains an infinite number of operators that do not correspond directly to any SUGRA modes. One class of such operators are the so called Konishi-like operators that may be defined as classically conserved currents of the non-interacting boundary CFT [3]. These operators correspond to massive string modes and hence cannot be seen in the SUGRA approximation of IIB string theory. Another class of boundary operators with no obvious SUGRA counterpart are multi-trace gauge invariant operators of $\mathcal{N}=4$ SYM. They can be protected or non-protected by superconformal invariance [4]. After some initial confusion regarding their status (see e.g. [5]), it was realized that such operators should have a SUGRA realization since they arise in the operator product expansions (OPEs) of the boundary operators as strong coupling [6, 7]. The general perception is that they correspond to "multi-particle" supergravity states [8], nevertheless we feel that their status is not yet fully clarified.

The study of boundary multi-trace operators can be accomplished *indirectly* through the operator product expansion of single-trace operators. Such studies have lead to a number of non-trivial checks for AdS/CFT [6, 7, 8, 9], as well as to some unexpected new properties [10]. One of them is the existence of operators whose scaling dimensions are non-renormalized despite the fact that they are not in the BPS list [6, 7]. A new protection mechanism had to be invoked to explain this phenomenon [11]. Also, it was demonstrated that multi-trace operators can acquire finite non-zero anomalous dimensions at strong coupling, but the detailed mechanism of such a phenomenon remains unclear.

Recently the interest on multi-trace operators has been revived due to ideas coming from string theory. Namely, it has been argued that the "multi-particle" SUGRA states might correspond to the "remnants" of non-local string couplings [12]. Such ideas give rise to the practical question of how explicit calculations of correlators involving multi-trace operators can be done in the context of AdS/CFT. This question is in direct relevance to the study of multi-trace deformations of the boundary CFT. An answer to this question came with the works of [13] and [14], where it has been argued that multi-trace deformations of the boundary CFT can by studied via AdS/CFT by a generalization of the boundary conditions. Further refinement was proposed in [15] and [16] such that both regular and irregular boundary conditions are properly taken into account.

In this letter we approach the problem of incorporating multi-trace deformations, directly from the boundary CFT side. For non-zero sources the standard AdS/CFT correspondence may be viewed as a prescription for deforming the strongly coupled boundary CFT by single-trace operators coupled to x-dependent couplings. When the boundary effective action is evaluated at least up to quartic order in the sources, it is possible that the operators which appear in the OPE of the single-traces might contribute to the deformation of the boundary CFT. In explicit AdS/CFT examples such operators in general include a number of multi-trace operators which may be relevant, marginal or irrelevant. Only the relevant and the marginal ones survive in the UV limit. To illustrate our approach, we study here a simple example of a scalar field in AdS_5 with cubic self interaction [17]. By an appropriate reparametrization of the x-dependent couplings (sources) we arrange so that the boundary CFT is perturbed by marginal operators. Moreover, we show that the boundary conformal anomaly plays the role of the bare coupling for such marginal deformations. Our methods can be applied to study the double-trace deformation in examples of AdS/CFT where the boundary four-point functions and OPEs are known, such as $\mathcal{N}=4$ SYM in $d=4$ or the (2,0) tensor multiplet in $d=6$[1] [18].

2. Multi-trace deformations and boundary conditions

The standard procedure for calculating correlations functions in a boundary CFT from AdS/CFT correspondence may be schematically written as

$$Z_{\rm SUGRA}[\phi_0] = \int (\mathcal{D}\phi,\phi_0) e^{-S_{\rm gr}[\phi]} \equiv e^{W_R[\phi_0]} \equiv \langle e^{\int \phi_0 \mathcal{O}} \rangle_{\rm CFT} \,. \tag{2.1}$$

In words this reads that performing the SUGRA path integral with prescribed boundary conditions ϕ_0 on the fields ϕ yields the generating functional (effective action) $W_R[\phi_0]$ of

[1] Of course, there is no notion of single- or multi-trace operators in the (2,0) tensor multiplet, nevertheless one can still apply OPE techniques to study the operator content of the theory at strong coupling [18].

connected renormalized correlation functions of the operator $\mathcal{O}$ in the boundary CFT. The last equality in (2.1) shows that one may also view the AdS/CFT correspondence as a definite prescription to deform the boundary CFT by the operator $\mathcal{O}$ coupled to the x-dependent coupling ϕ_0. This latter interpretation may not be widely known, nevertheless there exist extensive studies of CFT deformations by x-dependent couplings [21] and their general results can be applied to AdS/CFT.

The procedure described by (2.1) shows the difficulties with multi-trace deformations; since in explicit SUGRA calculations the fields ϕ are sources for single-trace boundary operators one does not know how to calculate directly correlation functions of multi-trace operators, or equivalently how to perturb the boundary CFT by multi-trace deformations. A procedure to incorporate multi-trace operators has been recently proposed in [13, 14]. The solution to the second order bulk equations of motion[2] for the SUGRA field ϕ behaves in general near the AdS_{d+1} conformal boundary $r = 0$ as

$$\phi(r,\bar{x})|_{r\to 0} = r^{d-\Delta}[\phi_0(\bar{x}) + O(r^2)] + r^{\Delta}[A(\bar{x}) + O(r^2)]\,, \qquad \Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4m^2}\,, \tag{2.2}$$

where $m^2 \geq -d^2/4$ is the AdS mass. The parameter Δ becomes the scaling dimension of the boundary operator $\mathcal{O}$.[3] To calculate now the expectation value $\langle \exp[\int \phi_0 \mathcal{O}]\rangle$ in the boundary CFT one needs the leading behaviour of $\phi(r,\bar{x})$ near the boundary which is given by the first term in (2.2). Nevertheless, since it has been shown [19] that $A(\bar{x})$ in (2.2) corresponds to the expectation value of the operator $\mathcal{O}$, the result of the expectation value calculation above can be schematically represented as $\exp[\int \phi_0 A]$. Now, the proposal of [14] is that if one wants to calculate the expectation value $\langle \exp[\int \hat{\phi}_0 \mathcal{O} + \int \mathcal{F}(\mathcal{O}, \mathrm{d}\mathcal{O})]\rangle$, where $\mathcal{F}(\mathcal{O}, \mathrm{d}\mathcal{O})$ is a generic functional of the operator $\mathcal{O}$, then one should solve the same bulk equations as the ones leading to (2.2) but with modified boundary conditions such that the field ϕ now behaves at the boundary as

$$\phi(r,\bar{x})|_{r=0} = \hat{\phi}_0(\bar{x}) + \frac{\delta \mathcal{F}(A, \mathrm{d}A)}{\delta A(\bar{x})}\,. \tag{2.3}$$

Then, $\hat{\phi}_0$ in (2.3) provides the source to the boundary operator $\mathcal{O}$ and the remaining term the source to the multi-trace operator represented by the functional $\mathcal{F}(\mathcal{O}, \mathrm{d}\mathcal{O})$. Refinements of the above proposal to include a proper treatment of both the so called regular and irregular boundary modes were presented in [15, 16].

3. The boundary action to quartic order

The proposal for incorporating multi-trace deformations of the boundary CFT by modifying the boundary conditions can in principle be used to perform explicit calculations in specific AdS/CFT models. As we mentioned in the introduction, an alternative way of

[2] We use throughout the euclidean version of the Poincaré patch of AdS_{d+1} where $\mathrm{d}x^\mu \mathrm{d}x_\mu = \frac{1}{r^2}(\mathrm{d}r^2 + \mathrm{d}\bar{x}^2)$ with $x = (r, \bar{x})$.

[3] We assume here that $\Delta \geq d/2$ as is relevant for our model as well as for the the standard IIB/$\mathcal{N} = 4$ SYM duality.

JHEP06(2002)009

incorporating multi-trace operators in the boundary effective action could be through the OPE of single-trace operators. This entails the calculation of the boundary effective action up to quartic order in the sources to read off the four-point functions of the single-trace operators. One advantage of this approach is, nevertheless, that one can use the known results for the four-point functions of some boundary CFTs from AdS/CFT correspondence, such as $\mathcal{N}=4$ SYM to study their corresponding boundary deformations. The purpose of this work is to illustrate our method in the toy model of a scalar field in AdS_5 with cubic self-interaction [17]. The classical massive action is

$$S_{\mathrm{gr}}[\phi]=\frac{1}{2}\int \mathrm{d}^5x\sqrt{g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+m^2\phi^2)+\frac{\lambda}{3!}\int \mathrm{d}^5x\sqrt{g}\phi^3\,, \tag{3.1}$$

where λ is an AdS coupling constant. The equations of motion are

$$(\nabla^2-m^2)\phi=\frac{\lambda}{2}\phi^2\,. \tag{3.2}$$

One needs to solve the equations of motion (3.2) subject to boundary conditions imposed at $r=0$ and substitute their solution back to (3.1). This way one obtains a functional of the boundary conditions which is interpreted as the generating functional for connected renormalized correlation functions of the boundary CFT. A nice way to accomplish that is to first solve an intermediate problem which amounts to exactly the same procedure as described above but with the boundary conditions now imposed on some hypersurface of AdS_5 *near* the boundary as

$$\phi(r,\bar{x})|_{r=\epsilon}=\phi_\epsilon(\bar{x})\,,\qquad \epsilon\ll 1\,. \tag{3.3}$$

Solving now the equations of motion (3.2) with the boundary condition (3.3) and substituting their solution back into (3.1) we will get a functional of ϕ_ϵ. The latter is then interpreted as the *regularized* generating functional of the boundary CFT. In doing the above, one must always keep in mind that the r integration should also be restricted to the range $r\in[\epsilon,\infty)$.

The procedure above has been described in a number of works and here we recapitulate its essential points. The general solution of the non-homogeneous equation of motion (3.2) with the boundary condition (3.3) is

$$\phi(r,\bar{x})=\bar{\phi}(r,\bar{x})+\frac{\lambda}{2}\int \mathrm{d}^5y\sqrt{g}G_\epsilon(x,y)\phi^2(y)\,, \tag{3.4}$$

where $\bar{\phi}(r,\bar{x})$ is a solution of the homogeneous part of (3.2) that satisfies the boundary condition (3.3) and $G_\epsilon(x,y)$ is a Green's function of the homogeneous part of (3.2) that vanishes when either of its arguments lies on the "boundary" $r=\epsilon$. The latter can be written as

$$G_\epsilon(x,y)=G(x,y)+F(x,y)\,, \tag{3.5}$$

where

$$\begin{aligned}(\nabla^2-m^2)G(x,y)&=\delta^d(x-y)\,,\\(\nabla^2-m^2)F(x,y)&=0\,,\\F(x,y)|_{x,y\in\partial_\epsilon}&=-G(x,y)|_{x,y\in\partial_\epsilon}\,,\end{aligned} \tag{3.6}$$

and ∂_ϵ denotes the "boundary" surface $r = \epsilon$ of AdS_5. Explicit expression for the above quantities exist since the early days of AdS/CFT [20] and although they are not needed for our calculations here we give them for completeness in the appendix. We only present here the so called bulk-to-boundary propagator that allows one to reconstruct the bulk field given the boundary condition (3.3) as

$$\phi(r,\bar{x}) = \int \mathrm{d}^4\bar{x}' K(r;\bar{x},\epsilon;\bar{x}')\phi_\epsilon(\bar{x}')\,, \tag{3.7}$$

$$K(r;\bar{x},\epsilon;\bar{x}') = \left(\frac{r}{\epsilon}\right)^2 \int \frac{\mathrm{d}^4\bar{p}}{(2\pi)^4} e^{-\mathrm{i}\bar{p}(\bar{x}-\bar{x}')} \frac{\mathrm{K}_\alpha(|\bar{p}|r)}{\mathrm{K}_\alpha(|\bar{p}|\epsilon)}\,, \qquad \alpha = \Delta - 2\,, \tag{3.8}$$

where $\mathrm{K}_\alpha(z)$ are the standard modified Bessel functions. It is important to note that in writing (3.7) and (3.8) we tacitly imposed the condition that the bulk scalar field vanishes as $r \to \infty$. Substituting the above into the action (3.1) one obtains up to quartic order in the boundary condition

$$\begin{aligned} S[\phi_\epsilon] &= -\frac{1}{2}\int \mathrm{d}^4\bar{x}\,\epsilon^{-3}\phi_\epsilon(\bar{x})\partial_r\phi(r,\bar{x})|_{r=\epsilon} + \\ &\quad +\frac{\lambda}{3!}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\,\phi_\epsilon(\bar{x}_1)\phi_\epsilon(\bar{x}_2)\phi_\epsilon(\bar{x}_3)\Pi_{3,\epsilon}(\bar{x}_1,\bar{x}_2,\bar{x}_3) + \\ &\quad +\frac{\lambda^2}{8}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\mathrm{d}^4\bar{x}_4\,\phi_\epsilon(\bar{x}_1)\phi_\epsilon(\bar{x}_2)\phi_\epsilon(\bar{x}_3)\phi_\epsilon(\bar{x}_4)\times \\ &\qquad\qquad \times\Pi_{4,\epsilon}(\bar{x}_1,\bar{x}_2,\bar{x}_3,\bar{x}_4)\,, \end{aligned} \tag{3.9}$$

$$\Pi_{3,\epsilon}(\bar{x}_1,\bar{x}_2,\bar{x}_3) = \int_\epsilon^\infty \mathrm{d}r r^{-5}\int \mathrm{d}^4\bar{x} K(r;\bar{x},\epsilon;\bar{x}_1)K(r;\bar{x},\epsilon;\bar{x}_2)K(r;\bar{x},\epsilon;\bar{x}_3)\,, \tag{3.10}$$

$$\begin{aligned} \Pi_{4,\epsilon}(\bar{x}_1,\bar{x}_2,\bar{x}_3,\bar{x}_4) &= \int_\epsilon^\infty \mathrm{d}r\mathrm{d}r'(rr')^{-5}\int \mathrm{d}^4\bar{x}\mathrm{d}^4\bar{y}\Big[K(r;\bar{x},\epsilon;\bar{x}_1)K(r;\bar{x},\epsilon;\bar{x}_2)G_\epsilon(x,y)\times \\ &\qquad \times K(r';\bar{y},\epsilon;\bar{x}_3)K(r';\bar{y},\epsilon;\bar{x}_4)\Big]\,. \end{aligned} \tag{3.11}$$

From (3.7) and (3.8) one easily obtains the asymptotic behaviour of ϕ_ϵ near $\epsilon \to 0$ as

$$\phi_\epsilon(\bar{x})|_{\epsilon\to 0} = \epsilon^{4-\Delta}[\phi_0(\bar{x}) + O(\epsilon^2)] + \epsilon^\Delta[A(\bar{x}) + O(\epsilon^2)]\,. \tag{3.12}$$

The functions $\phi_0(\bar{x})$ and $A(\bar{x})$ would be two independent boundary data necessary for the complete solution of the second order bulk equation of motion (3.2), nevertheless due to the imposed regularity of the bulk solution implied by (3.8) there is a relationship between them as

$$A(\bar{x}) = C_\Delta \int \mathrm{d}^4\bar{x}' \frac{\phi_0(\bar{x}')}{(\bar{x}-\bar{x}')^{2\Delta}}\,, \qquad C_\Delta = \frac{\Gamma(\Delta)}{\pi^2\Gamma(\Delta-2)}\,. \tag{3.13}$$

For $r = 0$ the boundary action is a functional of the boundary data ϕ_0. For example, to quadratic order in ϕ_0 one finds

$$S_\epsilon[\phi_0] = \frac{1}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\Pi(\bar{x}_{12},\epsilon)\,, \tag{3.14}$$

$$\Pi(\bar{x}_{12},\epsilon) = \epsilon^{4-2\Delta}\int \frac{\mathrm{d}^4\bar{p}}{(2\pi)^4} e^{-\mathrm{i}\bar{p}\bar{x}_{12}}\left[(4-\Delta) - |\bar{p}|\epsilon\frac{\mathrm{K}_{\alpha-1}(|\bar{p}|\epsilon)}{\mathrm{K}_\alpha(|\bar{p}|\epsilon)}\right]. \tag{3.15}$$

The action (3.14) is interpreted as the regularized action of the boundary CFT since in general it contains a finite number of divergent terms as $\epsilon \to 0$. One can subtract these divergences by introducing local counterterms built out from the field $\phi(r,\bar{x})$ at $r=\epsilon$. This subtraction procedure amounts to a specific choice of renormalization scheme as the counterterms include finite parts [24]. After subtraction, one can take the limit $\epsilon \to 0$ and is left with an action that is the generating functional $W_R[\phi_0]$ of connected renormalized correlation functions of the operator $\mathcal{O}$. However, in order to apply the OPE we need the *partition function* of the boundary CFT which is the generating functional of *both* the connected and disconnected n-point functions. Up to four-point functions this reads

$$\begin{aligned} Z_R[\phi_0] &\equiv e^{W_R[\phi_0]} \\ &= 1+\frac{1}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R - \\ &\quad -\frac{\lambda}{3!}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\phi_0(\bar{x}_3)\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\rangle_R + \\ &\quad +\frac{\lambda^2}{4!}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\mathrm{d}^4\bar{x}_4\phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\phi_0(\bar{x}_3)\phi_0(\bar{x}_4)\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_R\,, \end{aligned} \tag{3.16}$$

where the symmetry factors have been changed according to the requirement that the correlation functions are totally symmetric with respect to permutations of their arguments. For completeness, the explicit formulae for the various correlators in (3.16) are given in the appendix and we refer the reader to [17] for the calculations of the integrals.[4]

If one has managed to remove completely all scale dependence from (3.16), then one would find the generating functional of a quantum CFT.[5] This is in general possible, however for special values of the dimension Δ one cannot do this. As it was shown in [22] following the ideas of [23], for integer values of the parameter α the renormalized correlation functions in (3.16) necessarily contain logarithms which break conformal invariance. The relevant case for our present work is when

$$\Delta = 2+k\,, \qquad k=0,1,2,\ldots \tag{3.17}$$

Then, the renormalized two-point function

$$\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R = C_{\mathcal{O}}\left[\frac{1}{\bar{x}_{12}^{2\Delta}}\right]_R\,, \tag{3.18}$$

contains logarithms and this implies the existence of a conformal anomaly in the theory as [22]

$$\langle\Theta\rangle \equiv \int \mathrm{d}^4\bar{x}\langle T_{\mu\mu}(\bar{x})\rangle = \frac{1}{2}\mathcal{P}_k\int \mathrm{d}^4\bar{x}\,\partial^{2k}\,\phi_0^2(\bar{x})\,, \qquad \mathcal{P}_k = C_{\mathcal{O}}\frac{2\pi^2}{2^{2k}\Gamma(k+1)\Gamma(k+2)}\,. \tag{3.19}$$

[4] Despite that fact that a complete analysis of the counterterms needed to renormalized the $\epsilon \to 0$ singularities in n-point functions for $n \geq 3$ has not been performed, one can obtain explicit expressions for non-coincident arguments by simply taking the $\epsilon \to 0$ limit of (3.8), (3.10) and (3.11) before performing the integrals.

[5] Notice that logarithms of the invariant ratios in four-point functions are in perfect agreement with conformal invariance since they do not need an arbitrary mass scale for their proper definition.

For example, when $\Delta = 2$ using the correct normalization for the two-point function $C_{\mathcal{O}} = 1/2\pi^2$ we have

$$\langle \Theta \rangle = \frac{1}{2} \int \mathrm{d}^4\bar{x}\, \phi_0^2(\bar{x})\,. \tag{3.20}$$

In this case only the two-point function contributes to the conformal anomaly.

4. Operator deformations and the OPE

The essential observation now is that once the renormalized effective action of the boundary CFT is calculated up to quartic order in the sources, operators that appear in the OPE of two $\mathcal{O}$'s are naturally incorporated into it. In the case of $\mathcal{N} = 4$ SYM such operators will in general include both single- as well as double-traces. In the simple model (3.1), the OPE analysis of the four-point function in (3.16) has been done in whole generality in [17] and here we summarise the essential points. The OPE of the fields $\mathcal{O}$ that reproduces the explicit form of the four-point function in (3.16) can be schematically written as

$$\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2) = \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \sum_{\{O\}} \frac{G_O}{C_O} \frac{1}{(\bar{x}_{12}^2)^{\Delta - \frac{\Delta_O}{2}}} \mathcal{C}(\bar{x}_{12}, \partial_{\bar{x}_2}) \cdot O(\bar{x}_2)\,, \tag{4.1}$$

where $\{O\}$ is an infinite set of operators with corresponding dimensions Δ_0 and even spin, while the dot in (4.1) denotes the appropriate tensor contraction. The OPE coefficients $\mathcal{C}(\bar{x}_{12}, \partial_{\bar{x}_2})$ are complicated non-local expression which are explicitly known. For example, when O is a scalar we have

$$\begin{aligned}
\mathcal{C}_O(\bar{x}_{12}, \partial_{\bar{x}_2}) &= \frac{1}{B(\frac{\Delta}{2}, \frac{\Delta}{2})} \int_0^1 \mathrm{d}t [t(1-t]^{\frac{\Delta}{2}-1} \times \\
&\quad \times \sum_{m=0}^{\infty} \frac{(-1)^m \Gamma(\Delta + 1 - \frac{d}{2})}{m! \Gamma(\Delta + 1 + m - \frac{d}{2})} \left[\frac{\bar{x}_{12}^2}{4} t(1-t) \right]^m \partial_{\bar{x}_2}^{2m} e^{t\bar{x}_{12}\cdot\partial_{\bar{x}_2}} \\
&= 1 + \frac{1}{2}(\bar{x}_{12})_\mu \partial_{\bar{x}_2,\mu} + \frac{\Delta + 2}{8(\Delta + 1)} (\bar{x}_{12})_\mu (\bar{x}_{12})_\mu \partial_{\bar{x}_2,\mu} \partial_{\bar{x}_2,\nu} - \\
&\quad - \frac{\Delta}{16(\Delta + 1)(\Delta + 1 - \frac{d}{2})} (\bar{x}_{12}^2) \partial_{\bar{x}_2}^2 + \cdots\,.
\end{aligned} \tag{4.2}$$

The parameter G_O is the unique coupling constant in the three-point function $\langle O\mathcal{O}\mathcal{O}\rangle$ and C_O is the normalization constant of the two-point function $\langle OO\rangle$. The infinite set of operators $\{O\}$ are classified according to increasing powers of their dimension and spin. In our case, the first few operators that contribute singular and marginal terms as $\bar{x}_{12} \to 0$ in (4.1) are the operator $\mathcal{O}$ itself, a scalar operator with dimension $2\Delta + \lambda^2\gamma_*$ which we denote as $\mathcal{O}^2$ and the energy momentum tensor which is a spin-2 operator with dimension d. The quantity γ_* is the anomalous dimension of the operator $\mathcal{O}^2$ which can be explicitly calculated from the results in [17]. In general, an OPE such as (4.1) is supposed to hold when the two operators are close to each other. However, there is strong evidence (see e.g. [25]) that in CFTs the OPE is an analytic partial wave expansion, even for $D > 2$, and as such it can be inserted into the generating functional (3.16) yielding

$$
\begin{aligned}
Z_R[\phi_0] = 1 &+ \frac{1}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\, \phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\, \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R - \\
&- \frac{\lambda}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\, \phi_0(\bar{x}_1)\phi_0(\bar{x}_2)\phi_0(\bar{x}_3)\, \times \\
&\qquad \times \frac{G_{\mathcal{O}}}{C_{\mathcal{O}}}\frac{1}{(\bar{x}_{32}^2)^{\frac{\Delta}{2}}}\,\mathcal{C}(\bar{x}_{32}, \partial_{\bar{x}_2})\cdot \langle \mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_1)\rangle_R + \\
&+ \frac{\lambda^2}{4}\int \mathrm{d}^4\bar{x}_1\cdots \mathrm{d}^4\bar{x}_4\phi_0(\bar{x}_1)\cdots\phi_0(\bar{x}_4)\, \times \\
&\qquad \times \sum_{\{O\}}\frac{G_O}{C_O}\frac{1}{(\bar{x}_{43}^2)^{\Delta-\frac{\Delta_O}{2}}}\,\mathcal{C}(\bar{x}_{43}, \partial_{\bar{x}_3})\cdot \langle O(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \\
&+ \frac{\lambda^2}{8}\int \mathrm{d}^4\bar{x}_1\cdot \mathrm{d}^4\bar{x}_4\, \phi_0(\bar{x}_1)\cdot\phi_0(\bar{x}_4)\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R\langle \mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_R\,. \qquad (4.3)
\end{aligned}
$$

From (4.3) we see that when we view the AdS/CFT correspondence as a deformation of the boundary CFT with an x-dependent coupling constant $\phi_0(\bar{x})$, many operators enter naturally into the partition function via the OPE. Of course, keeping the x-dependent coupling non-zero in general breaks the conformal invariance. The meaning then of equations such as (4.3) is that conformal invariance is broken by perturbing the CFT by all possible operators that appear in the OPE (4.1) [26].

Given the form of the generating functional (4.3) it is possible to choose exactly which operator deforms the boundary CFT by appropriately adjusting the sources ϕ_0. Such an adjustment of the sources ϕ_0 has a dual interpretation; from the point of view of the bulk theory it corresponds to a modification of the boundary conditions (3.12) [13, 14], while from the point of view of the boundary theory it corresponds to a reparametrization of the x-dependent couplings. Namely, setting

$$
\phi_0(\bar{x}) = \hat{\phi}_0(\bar{x}) + \phi_1(\bar{x})\,, \qquad (4.4)
$$

we find from (4.3)

$$
\begin{aligned}
Z_R[\phi_0] = Z_R[\hat{\phi}_0] &+ \int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\phi_1(\bar{x}_1)\hat{\phi}_0(\bar{x}_2)\, \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R - \\
&- \frac{\lambda}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\, \phi_1(\bar{x}_1)\hat{\phi}_0(\bar{x}_2)\hat{\phi}_0(\bar{x}_3)\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\rangle_R + \cdots\,. \qquad (4.5)
\end{aligned}
$$

To proceed now we have to take into account an important point that follows from the analysis of [17]. As we have mentioned, the four-point function in (3.16) contains *both* connected and disconnected parts. In the normalization of (3.16) the disconnected part comes with a factor λ^{-2} in front. It has been observed in [17] that in generic AdS graphs the contribution to the four-point function from the operator $\mathcal{O}$ itself comes *only* from the

JHEP06(2002)009

connected part. This means that the form of the OPE (4.1) implies

$$
\begin{aligned}
Z[\hat{\phi}_0] = 1 &+ \frac{1}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\, \hat{\phi}_0(\bar{x}_1)\hat{\phi}_0(\bar{x}_2)\, \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R - \\
&- \frac{\lambda G_{\mathcal{O}}}{2C_\Delta}\int \mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\, \hat{\phi}_0(\bar{x}_2)\hat{\phi}_0(\bar{x}_3)\, \frac{1}{(\bar{x}_{32}^2)^{\frac{\Delta}{2}}}\, \mathcal{C}_{\mathcal{O}}(\bar{x}_{32},\partial_{\bar{x}_2})A(\bar{x}_2) + \\
&+ \frac{\lambda^2 G_{\mathcal{O}}}{4C_{\mathcal{O}}}\int \mathrm{d}^4\bar{x}_1\cdots\mathrm{d}^4\bar{x}_4\, \hat{\phi}_0(\bar{x}_1)\cdots\hat{\phi}_0(\bar{x}_4)\, \times \\
&\qquad\qquad \times \frac{1}{(\bar{x}_{43}^2)^{\frac{\Delta}{2}}}\, \mathcal{C}_{\mathcal{O}}(\bar{x}_{43},\partial_{\bar{x}_3})\langle \mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \\
&+ \frac{G_{\mathcal{O}^2}(\lambda)}{4C_{\mathcal{O}^2}(\lambda)}\int \mathrm{d}^4\bar{x}_1\cdots\mathrm{d}^4\bar{x}_4\hat{\phi}_0(\bar{x}_1)\cdots\hat{\phi}_0(\bar{x}_4)\, \times \\
&\qquad\qquad \times (\bar{x}_{34}^2)^{\frac{\lambda^2\gamma_*}{2}}\, \mathcal{C}_{\mathcal{O}^2}(\bar{x}_{43},\partial_{\bar{x}_3})\langle \mathcal{O}^2(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \\
&+ \frac{\lambda^2}{8}\int \mathrm{d}^4\bar{x}_1\cdots\mathrm{d}^4\bar{x}_4\hat{\phi}_0(\bar{x}_1)\cdots\hat{\phi}_0(\bar{x}_4)\, \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R\langle \mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_R + \cdots\,,
\end{aligned}
\tag{4.6}
$$

where the dots represent terms involving higher orders in λ and correlation functions involving tensor operators, while we have also used (3.13). The meaning of (4.6) is that the coupling and scaling dimension of the operator $\mathcal{O}^2$ contain both $O(\lambda^0)$ as well as $O(\lambda^2)$ contributions. This is generically true for all operators in the OPE (4.1) except $\mathcal{O}$ itself. We have seen already the λ-dependence for the scaling dimension of $\mathcal{O}^2$, while for the coupling (more precisely, the ratio of the coupling the the two-point function normalization), we may write

$$
\frac{G_{\mathcal{O}^2}(\lambda)}{C_{\mathcal{O}^2}(\lambda)} = \left.\frac{G_{\mathcal{O}^2}}{C_{\mathcal{O}^2}}\right|_0 [1+\lambda^2 b_* + O(\lambda^3)]\,, \tag{4.7}
$$

where the subscript 0 denotes the λ-independent part coming from the disconnected graphs in the four-point function. The finite number b_* can be read from the results in [17].

We now observe that if the two ϕ_1-dependent terms of (4.5) cancel the third and fourth terms in the rhs of (4.6), then $Z_R[\hat{\phi}_0]$ would generate correlation functions of $\mathcal{O}$ with $\mathcal{O}^2$ insertions. This can be arranged if the following integral equations are satisfied

$$
\frac{C_{\mathcal{O}}}{C_\Delta}\int \mathrm{d}^4\,\bar{x}\phi_1(\bar{x})A(\bar{x}) - \frac{\lambda G_{\mathcal{O}}}{2C_\Delta}\int \mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\, \hat{\phi}_0(\bar{x}_2)\hat{\phi}_0(\bar{x}_3)\, \frac{1}{(\bar{x}_{32}^2)^{\frac{\Delta}{2}}}\, \mathcal{C}_{\mathcal{O}}(\bar{x}_{32},\partial_{\bar{x}_2})A(\bar{x}_2) = 0\,, \tag{4.8}
$$

and

$$
\begin{aligned}
&\frac{\lambda}{2}\int \mathrm{d}^4\bar{x}_1\mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\phi_1(\bar{x}_1)\,\hat{\phi}_0(\bar{x}_2)\hat{\phi}_0(\bar{x}_3)\,\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\rangle_R - \\
&- \frac{\lambda^2 G_{\mathcal{O}}}{4C_{\mathcal{O}}}\int \mathrm{d}^4\bar{x}_1\cdots\mathrm{d}^4\bar{x}_4\hat{\phi}_0(\bar{x}_1)\cdots\hat{\phi}_0(\bar{x}_4)\frac{1}{(\bar{x}_{43}^2)^{\frac{\Delta}{2}}}\, \mathcal{C}_{\mathcal{O}}(\bar{x}_{43},\partial_{\bar{x}_3})\langle \mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R = 0\,.
\end{aligned}
\tag{4.9}
$$

From (4.8) we find

$$
\phi_1(\bar{x}) = \frac{\lambda G_{\mathcal{O}}}{2C_{\mathcal{O}}}\int \mathrm{d}^4\bar{x}_2\mathrm{d}^4\bar{x}_3\, \hat{\phi}_0(\bar{x}_2)\hat{\phi}_0(\bar{x}_3)\, \frac{1}{(\bar{x}_{32}^2)^{\frac{\Delta}{2}}}\, \mathcal{C}_{\mathcal{O}}(\bar{x}_{32},\partial_{\bar{x}_2})\delta^4(\bar{x}-\bar{x}_2)\,. \tag{4.10}
$$

Substituting (4.10) into (4.9) and integrating the delta function by parts we see that (4.9) is also satisfied. Therefore, by reparametrizing the initial sources as in (4.4) and (4.10) one gets a new partition function $Z_R[\hat{\phi}_0]$ that corresponds to a deformation of the original CFT by the operator $\mathcal{O}^2$. For example, the leading correction to the *connected* two-point function would then read

$$\begin{aligned}\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle'_R &= \langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \\ &+ \left.\frac{G_{\mathcal{O}^2}}{2C_{\mathcal{O}^2}}\right|_0 \int \mathrm{d}^4\bar{x}_3\mathrm{d}^4\bar{x}_4\, \hat{\phi}_0(\bar{x}_3)\hat{\phi}_0(\bar{x}_4)\left[1+\frac{\lambda^2\gamma_*}{2}\ln(\bar{x}_{34}^2\mu^2)+\lambda^2 b_*\right]\times \\ &\times \mathcal{C}_{\mathcal{O}^2}(\bar{x}_{43},\partial_{\bar{x}_3})\langle\mathcal{O}^2(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R+\cdots\,, \end{aligned} \tag{4.11}$$

where the arbitrary mass parameter μ is necessary for the correct definition of the logarithm. The dots in (4.11) correspond to terms of order $O(\lambda^4)$ as well as to three-point functions involving scalar operators with dimensions greater that 2Δ and operators with non-zero (even) spin [17]. Now, all angular dependent terms in the integrand on the rhs of (4.11) drop out. This means that one is left with three-point functions involving only scalar operators. Finally, we restrict ourselves to the case when $\Delta = 2$. Then, from translation invariance and renormalization arguments one obtains

$$\begin{aligned}\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle'_R &= \langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \left.\frac{G_{\mathcal{O}^2}}{2C_{\mathcal{O}^2}}\right|_0 \int \mathrm{d}^4\bar{x}_4\hat{\phi}_0^2(\bar{x}_4)\times \\ &\times\int \mathrm{d}^4\bar{x}_3\left[1+\frac{\lambda^2\gamma_*}{2}\ln(\bar{x}_3^2\mu^2)+\lambda^2 b_*\right]\langle\mathcal{O}^2(\bar{x}_3)\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R+\cdots \end{aligned} \tag{4.12}$$

where now the dots in (4.12) represent multiple $\mathcal{O}^2$ insertions coming from higher correlation functions. Insertions from scalar operators with dimensions greater than $2\Delta = 4$ are irrelevant and drop out. One way to see how (4.12) is derived from (4.11) is to introduce a short-distance cut-off $L \ll 1$ in (4.11) and write a simple representation for the x-dependent couplings as

$$\hat{\phi}_0(\bar{x}) = L^{-2}\int \mathrm{d}^4\bar{y}\,\delta^4(\bar{x}-\bar{y})\,. \tag{4.13}$$

Then, using translation invariance one can show that only the unit term of the OPE coefficient $\mathcal{C}_{\mathcal{O}^2}$ survives the UV limit $L\to 0$ of (4.11). Since now (4.12) is to be interpreted as the deformations of the CFT by a coupling of the form $\int\hat{\phi}_0\mathcal{O}^2$, the $O(\lambda^2)$ terms could be absorbed in a renormalization of the of the operator $\mathcal{O}^2$ as

$$\mathcal{O}^2(\bar{x}) = \left[1-\frac{\lambda^2\gamma_*}{2}\ln(\bar{x}^2\mu^2)-\lambda^2 b_*\right]\mathcal{O}^2_{\mathrm{ren}}(\bar{x})\,, \tag{4.14}$$

and then (4.12) becomes by virtue of (3.20)

$$\langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle'_R = \langle\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R + \left.\frac{G_{\mathcal{O}^2}}{C_{\mathcal{O}^2}}\right|_0 \langle\Theta\rangle\int \mathrm{d}^4\bar{x}\langle\mathcal{O}^2_{\mathrm{ren}}(\bar{x})\mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle_R+\cdots\,. \tag{4.15}$$

5. Conclusions and outlook

For the simple model of a scalar field in AdS_5 with cubic self-interaction we showed that deformations of the boundary CFT by scalar operators which appear in the OPE of the basic operators $\mathcal{O}$ can be incorporated in the context of AdS/CFT by an appropriate adjustment of the boundary data ϕ_0. From the point of view of the bulk theory such an adjustment should correspond to a change in the boundary condition for the bulk fields. From the point of view of the boundary theory such an adjustment is a reparametrization of the x-dependent coupling associated with the boundary operator $\mathcal{O}$. Our results can be generalized to the case when the OPE contains many more relevant and marginal operators.

Essential role in our calculations played the OPE of the boundary fields $\mathcal{O}$. In the $\mathcal{N}=4$ SYM case, the generalization of our results should provide a concrete method to study multi-trace deformations of the boundary CFT via OPEs of single-trace operators. Moreover, it is conceivable that our method could also be useful in studies of deformations of the (2,0) tensor multiplet in $d=6$. In particular, in the $\mathcal{N}=4$ SYM case one knows a list of double-trace operators in various representations of the SU(4) symmetry group that have naive dimension 4 and could in principle be incorporated by our method. Furthermore, their three-point function couplings and two-point function normalization constants are known. One has to distinguish between the various operators that are deforming the boundary CFT. For example, the double-trace operator in the [**1**] of SU(4), denoted $\mathcal{O}_{\mathbf{1}}$ in [6], acquires anomalous dimensions of order $1/N^2$ at strong coupling. Therefore, to incorporate such a deformation one needs to know the anomalous dimension of the operator as well as the correction to its coupling. On the other hand, the double-trace operator in the [**20**] of SU(4), denoted $\mathcal{O}_{\mathbf{20}}$ in [6], has protected dimension and can be incorporated more easily. It is highly probable that both operators above break the conformal invariance of the theory, although the case of $\mathcal{O}_{\mathbf{20}}$ deserves further study.

In explicit AdS/CFT calculations, bulk actions such as (3.1) do not in general include arbitrary parameters since the relative coefficients of all terms are fixed by SUGRA. Nevertheless, deforming the boundary CFT by a double-trace operator entails the introduction of an arbitrary "bare" coupling constant. As shown in (4.15), for marginal deformations this is nothing but the integrated conformal anomaly. In the simple model studied here, (3.19) is the only conformal anomaly in the boundary CFT, however in explicit SUGRA calculations we expect that the gravitational anomaly will also play a role as a bare coupling for double-trace marginal deformations. The implications of such a result for a better understanding of the double-trace operators in terms of string theory, either as non-local string couplings or otherwise, is an interesting question.

Acknowledgments

I would like to thank G. Arutyunov for very interesting discussions as well as a critical reading of the manuscript.

A. Appendix

We give for completeness the explicit representation for the bulk-to-bulk propagator needed for the calculations of the boundary effective action.

$$
\begin{aligned}
G(x,y) &= -c_\Delta \xi^{-\Delta}{}_2F_1\left(\frac{\Delta}{2}+\frac{1}{2},\frac{\Delta}{2};\Delta-1;\xi^{-2}\right) \\
\xi^2 &= \frac{r^2+r'^2+(\bar{x}-\bar{y})^2}{2rr'}, \qquad c_\Delta = \frac{\Gamma(\Delta)}{2^{\Delta+1}\pi^2\Gamma(\Delta-1)} \\
F(x,y) &= \int \frac{\mathrm{d}^4\bar{p}}{(2\pi)^4} e^{-\mathrm{i}\bar{p}(\bar{x}-\bar{y})}(rr')^2 \mathrm{K}_\alpha(|\bar{p}|r)\mathrm{K}_\alpha(|\bar{p}|r')\frac{\mathrm{I}_\alpha(|\bar{p}|\epsilon)}{\mathrm{K}_\alpha(|\bar{p}|\epsilon)} \\
\partial_r G_\epsilon(x,y)|_{r=\epsilon} &= -\epsilon^3 K(\epsilon;\bar{x},r';\bar{y})
\end{aligned}
$$

The explicit expression we use for the correlators appearing in (3.16) are (for non-coincident points and general d) [17]

$$
\begin{aligned}
\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle &= C_{\mathcal{O}}\frac{1}{\bar{x}_{12}^{2\Delta}} \\
\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\rangle &= \frac{1}{4\pi^d}\frac{\Gamma^3(\frac{\Delta}{2})\Gamma(\frac{3\Delta}{2}-\frac{d}{2})}{\Gamma^3(\Delta-\frac{d}{2})}\frac{1}{(\bar{x}_{12}^2\bar{x}_{13}^2\bar{x}_{23}^2)^{\frac{\Delta}{2}}} \\
\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle &= \frac{1}{\lambda^2}\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_{\mathrm{disc}} + \\
&\quad + \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_{\mathrm{conn}} \\
\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_{\mathrm{disc}} &= \langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\rangle\langle \mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle + (\bar{x}_2\leftrightarrow\bar{x}_3) + (\bar{x}_2\leftrightarrow\bar{x}_4) \\
\langle \mathcal{O}(\bar{x}_1)\mathcal{O}(\bar{x}_2)\mathcal{O}(\bar{x}_3)\mathcal{O}(\bar{x}_4)\rangle_{\mathrm{conn}} &= -\int_0^\infty \frac{\mathrm{d}r\mathrm{d}r'}{(rr')^{d+1}}\int \mathrm{d}^d\bar{x}\mathrm{d}^d\bar{y}\Big[\hat{K}(r;\bar{x},,\bar{x}_1)\hat{K}(r;\bar{x},\bar{x}_2)\times \\
&\quad \times G(x,y)\hat{K}(r';\bar{y},\bar{x}_3)\hat{K}(r';\bar{y},\bar{x}_4)\Big] + (\bar{x}_2\leftrightarrow\bar{x}_3) + (\bar{x}_2\leftrightarrow\bar{x}_4) \\
\hat{K}(r';\bar{y},\bar{x}) &= C_\Delta\left[\frac{r'}{r'^2+(\bar{y}-\bar{x})^2}\right]^\Delta
\end{aligned}
$$

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[2] S. Ferrara, C. Fronsdal and A. Zaffaroni, *On $N = 8$ supergravity on AdS_5 and $N = 4$ superconformal Yang-Mills theory*, *Nucl. Phys.* **B 532** (1998) 153 [hep-th/9802203].

[3] M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *Properties of the Konishi multiplet in $N = 4$ SYM theory*, *J. High Energy Phys.* **05** (2001) 042 [hep-th/0104016].

[4] L. Andrianopoli and S. Ferrara, *Short and long* SU(2, 2/4) *multiplets in the AdS/CFT correspondence*, *Lett. Math. Phys.* **48** (1999) 145 [hep-th/9812067].

JHEP06(2002)009

[5] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Comments on 4-point functions in the CFT/AdS correspondence*, *Phys. Lett.* **B 452** (1999) 61 [hep-th/9808006].

[6] G. Arutyunov, S. Frolov and A.C. Petkou, *Operator product expansion of the lowest weight CPOS in $N = 4$ SYM_4 at strong coupling*, *Nucl. Phys.* **B 586** (2000) 547 [hep-th/0005182].

[7] G. Arutyunov, S. Frolov and A. Petkou, *Perturbative and instanton corrections to the OPE of CPOS in $N = 4$ SYM_4*, *Nucl. Phys.* **B 602** (2001) 238 [hep-th/0010137].

[8] E. D'Hoker, S.D. Mathur, A. Matusis and L. Rastelli, *The operator product expansion of $N = 4$ SYM and the 4-point functions of supergravity*, *Nucl. Phys.* **B 589** (2000) 38 [hep-th/9911222].

[9] B. Eden, P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Simplifications of four-point functions in $N = 4$ supersymmetric Yang-Mills theory at two loops*, *Phys. Lett.* **B 466** (1999) 20 [hep-th/9906051];
M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *On the logarithmic behavior in $N = 4$ SYM theory*, *J. High Energy Phys.* **08** (1999) 020 [hep-th/9906188];
B. Eden, C. Schubert and E. Sokatchev, *Three-loop four-point correlator in $N = 4$ SYM*, *Phys. Lett.* **B 482** (2000) 309 [hep-th/0003096];
M. Bianchi, S. Kovacs, G. Rossi and Y.S. Stanev, *Anomalous dimensions in $N = 4$ SYM theory at order g^4*, *Nucl. Phys.* **B 584** (2000) 216 [hep-th/0003203];
S. Penati, A. Santambrogio and D. Zanon, *More on correlators and contact terms in $N = 4$ SYM at order g^4*, *Nucl. Phys.* **B 593** (2001) 651 [hep-th/0005223];
S. Penati and A. Santambrogio, *Superspace approach to anomalous dimensions in $N = 4$ SYM*, *Nucl. Phys.* **B 614** (2001) 367 [hep-th/0107071].

[10] B. Eden, A.C. Petkou, C. Schubert and E. Sokatchev, *Partial non-renormalisation of the stress-tensor four-point function in $N = 4$ SYM and AdS/CFT*, *Nucl. Phys.* **B 607** (2001) 191 [hep-th/0009106].

[11] G. Arutyunov, B. Eden and E. Sokatchev, *On non-renormalization and ope in superconformal field theories*, *Nucl. Phys.* **B 619** (2001) 359 [hep-th/0105254].

[12] O. Aharony, M. Berkooz and E. Silverstein, *Multiple-trace operators and non-local string theories*, *J. High Energy Phys.* **08** (2001) 006 [hep-th/0105309];
O. Aharony, M. Berkooz and E. Silverstein, *Non-local string theories on $AdS_3 \times S^3$ and stable non-supersymmetric backgrounds*, *Phys. Rev.* **D 65** (2002) 106007 [hep-th/0112178].

[13] M. Berkooz, A. Sever and A. Shomer, *Double-trace deformations, boundary conditions and spacetime singularities*, *J. High Energy Phys.* **05** (2002) 034 [hep-th/0112264].

[14] E. Witten, *Multi-trace operators, boundary conditions and AdS/CFT correspondence*, hep-th/0112258.

[15] W. Mück, *An improved correspondence formula for AdS/CFT with multi-trace operators*, *Phys. Lett.* **B 531** (2002) 301 [hep-th/0201100].

[16] P. Minces, *Multi-trace operators and the generalized AdS/CFT prescription*, hep-th/0201172.

[17] L. Hoffmann, A.C. Petkou and W. Rühl, *Aspects of the conformal operator product expansion in AdS/CFT correspondence*, *Adv. Theor. Math. Phys.* **4** (2002) 571 [hep-th/0002154].

[18] G. Arutyunov and S. Frolov, *Four-point functions of lowest weight CPOS in $N = 4$ SYM_4 in supergravity approximation*, *Phys. Rev.* **D 62** (2000) 064016 [hep-th/0002170];
G. Arutyunov and E. Sokatchev, *Implications of superconformal symmetry for interacting (2,0) tensor multiplets*, hep-th/0201145.

JHEP06(2002)009

[19] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104].

[20] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the CFT_d/AdS_{d+1} correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058];
W. Mück and K.S. Viswanathan, *Conformal field theory correlators from classical scalar field theory on AdS_{d+1}*, *Phys. Rev.* **D 58** (1998) 041901 [hep-th/9804035].

[21] H. Osborn, *Weyl consistency conditions and a local renormalization group equation for general renormalizable field theories*, *Nucl. Phys.* **B 363** (1991) 486.

[22] A. Petkou and K. Skenderis, *A non-renormalization theorem for conformal anomalies*, *Nucl. Phys.* **B 561** (1999) 100 [hep-th/9906030].

[23] H. Osborn and A.C. Petkou, *Implications of conformal invariance in field theories for general dimensions*, *Ann. Phys. (NY)* **231** (1994) 311 [hep-th/9307010].

[24] G.E. Arutyunov and S.A. Frolov, *On the origin of supergravity boundary terms in the AdS/CFT correspondence*, *Nucl. Phys.* **B 544** (1999) 576 [hep-th/9806216];
S. de Haro, S.N. Solodukhin and K. Skenderis, *Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence*, *Commun. Math. Phys.* **217** (2001) 595 [hep-th/0002230].

[25] E.S. Fradkin and M.Y. Palchik, *Conformal quantum field theory in D-dimensions*, Kluwer, Dordrecht, Netherlands 1996.

[26] J.L. Cardy, *Scaling and renormalization in statistical physics*, Cambridge University Press, Cambridge 1997.

JHEP06(2002)009